

MATH 134A Review: Expectation and Variance

Facts to Know

Let X be a (discrete) random variable with probability distribution function $p(x) = \mathbb{P}(X = x)$.

$$X = x_1, x_2, x_3, \dots$$

The expectation or “center” of X is

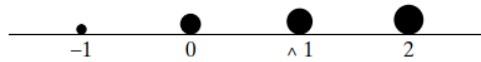
$$\mathbb{E}(X) = \sum_x xp(x) = x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + \dots$$

Let $\mu = \mathbb{E}(X)$. The variance or “spread” of X about μ is

$$\begin{aligned}\mathbb{E}((X - \mu)^2) &= \sum_x (x - \mu)^2 \cdot p(x) \\ &= (x_1 - \mu)^2 p(x_1) + (x_2 - \mu)^2 p(x_2) + \dots\end{aligned}$$

Examples

A random variable X may take values of either $-1, 0, 1, 2$ with probabilities $0.10, 0.25, 0.30, 0.35$, respectively.



1. Find the expectation of X .

$$\begin{aligned}\mathbb{E}(X) = \mu &= (-1)(0.1) + (0)(0.25) + (1)(0.30) + (2)(0.35) \\ &= -0.1 + 0 + \underbrace{0.3 + 0.7}_1 = 0.9\end{aligned}$$

2. Find the variance of X .

$$\begin{aligned}\mathbb{E}((X - \mu)^2) = s^2 &= (-1 - \mu)^2 (0.1) + (0 - \mu)^2 (0.25) + (1 - \mu)^2 (0.30) \\ &\quad + (2 - \mu)^2 (0.35) \\ &= (-1.9)^2 (0.1) + (0.9)^2 (0.25) + (0.1)^2 (0.3) \\ &\quad + (1.1)^2 (0.35) \geq 0\end{aligned}$$